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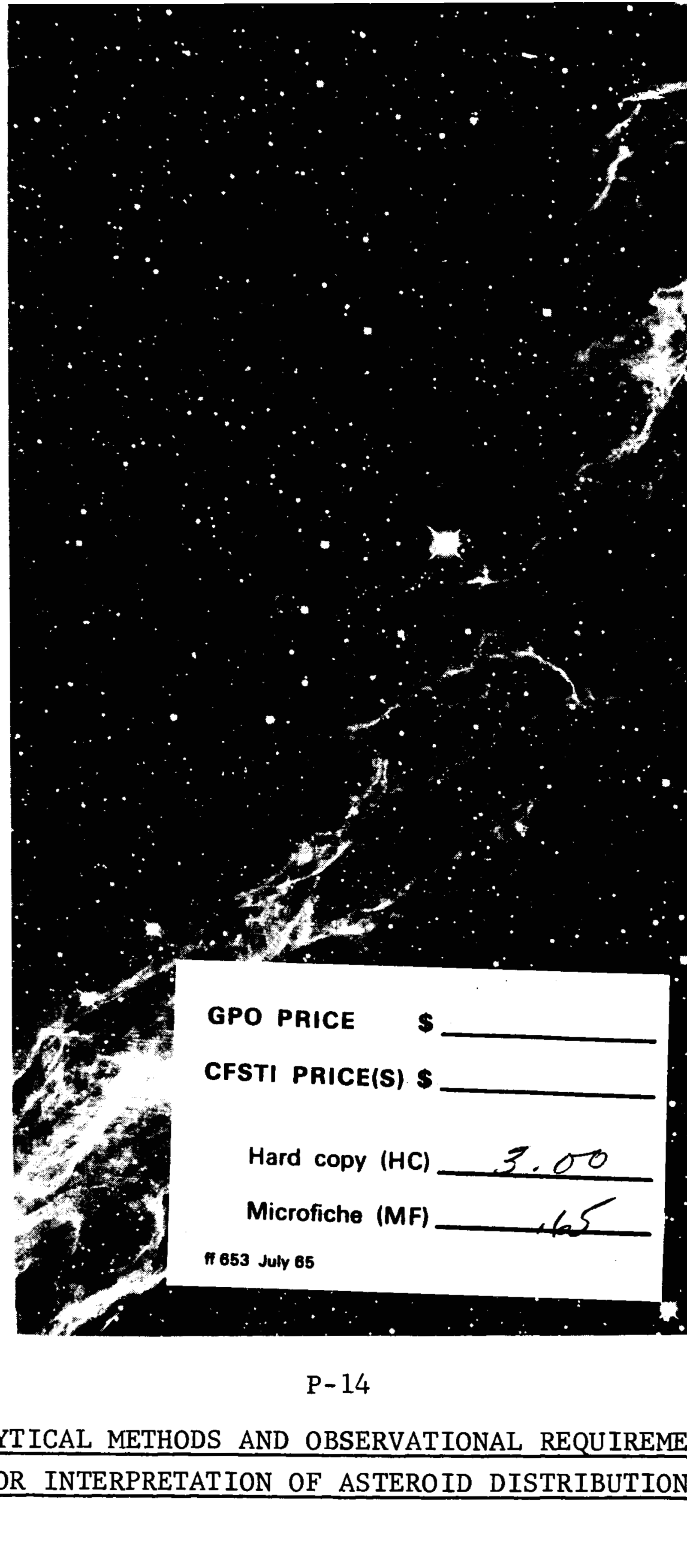

P-14

ANALYTICAL METHODS AND OBSERVATIONAL REQUIREMENTS FOR INTERPRETATION OF ASTEROID DISTRIBUTION

Report No. P-14

ANALYTICAL METHODS AND OBSERVATIONAL REQUIREMENTS FOR INTERPRETATION OF ASTEROID DISTRIBUTIONS

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SUMMARY

In this report, the significance of the distribution of the asteroids is considered. Questions are raised whether ordering mechanisms exist capable of arranging the asteroids in some identifiable distribution, or whether the asteroidal material is more or less randomly distributed throughout an essentially toroidal ring extending from Mars to Jupiter.

The conclusion is reached that the planets, in particular Jupiter, exert small perturbing forces which over long periods of time produce distinctive distributional features. These features are deterministic, and through suitable interpretation, the mechanical history of solar system events may possibly be traced back through time and contribute to the understanding of the solar system origins. The deeper understanding of distributional features can also be of value for the prediction of possible hazards to interplanetary space missions.

The purpose of this report is twofold: To outline general methods of analysis for the interpretation of asteroid observational data and their significance, and to recommend further observations to extend and support existing hypotheses and theoretical approaches. A summary list of recommended

observations, from both space probes and Earth based, is presented in the following table.

TABLE OF RECOMMENDED OBSERVATIONS

Observation	Specific Requirements	Purpose
Number and size distribution (in ecliptic plane)	1) In neighborhood of a libration point	Determine possible clustering Test hypotheses for stability analyses
	2) In neighborhood of a resonance with Jupiter period	
	a) Inner & central regions of asteroid ring	Test hypotheses and methods of celestial mechanics that now predict V-shaped numl distribution
	b) Outer regions of ring	Determine if clustering occurs and the extent of influence of higher order commensurabilities
	3) Through a continuous range of smaller sizes	Test comminution laws and establish semiempirical coefficients for fragmentation processes
Distributions out of the ecliptic plane	1) Orbital eccentricities	Test evolution hypotheses regarding plasma condensation processes
	2) Orbital inclinations	Test poorly convergent perturbation techniques for secular variations of higher inclination orbital elements Determine dispersion and relative velocities of eroding particles from asteroid surfaces
	3) Motion of perihelion	Understand motion of apse line for highe inclination orbits; also of significance to motion of comet orbits and relation to meteor streams
Orbital elements	1) For smaller members of recognized families	Provide a more adequate statistical mode Correlate spread in parametric values with the initial collision scatter velocities to provide estimates of family age

Observation	Specific Requirements	Purpose
		Permit determination of proper values for families
	2) More accurate measurements of members of a selected family	Extend validity of computations of secular variations over greater time interval
Orbital directions		Provide evidence relevant to cometary origins
Axial rotation and direction		Evidence of origins: collision fragments or primordial condensations
Surface physical condition	1) Reflectivity	Correlation with Earth-based observations and uniformity indicates non-fragment
	2) Roughness	Indicates collision fragments
	3) Erosion	Age indicator
	4) Craters	Impact history
Material physical and chemical properties		Provide impact parameters for collision mechanics
		Comparison with recovered meteorites to establish meteorite origins
		Comparison with comet core
		Age estimates
		Collision history

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1. INTRODUCTION

The asteroids, or minor planets, constitute a numerous group of bodies, which with few exceptions revolve in orbits of small eccentricity and inclination about the Sun in the region between Mars and Jupiter. The exceptions, usually with highly eccentric orbits, approach to within the orbits of Earth or Venus or recede to beyond Jupiter. The asteroids are solid bodies, of density similar to the Earth's crust, without atmospheres, having sizes ranging downward from about 770 km for Ceres presumably to micron size particles. Some are irregular in shape and rotate about their own axes with periods usually under 12 hr. Of the estimated 30,000 asteroids within reach of the large reflecting telescopes, reliable orbits have been calculated for about 1600.

The principal source of data is the list of asteroids, Ephemerides of Minor Planets, published yearly by the Institute of Theoretical Astronomy at Leningrad. This list contains the current orbital elements and magnitudes of what now number some 1650 asteroids. New discoveries and corrections are communicated in the Minor Planet Circulars issued by the University of Cincinnati Observatory. A study by Kuiper (1958) indicates that all 437 asteroids with photographic magnitude at opposition equal to 14 or less are probably known; and from a statistical extrapolation, there are estimated to be 33,600 asteroids of photographic magnitude brighter than 19.5. Estimates of the asteroid sizes are based upon measurements of their brightness; for absolute magnitudes from 4 to 8 the mean diameter is about 300 km; for the smaller observable sizes down to about 15 km, the magnitude ranges down to 12 or 13.

The asteroids are not randomly distributed throughout the belt but possess definite distributional features. Gaps in the distribution of asteroid orbits were recognized in 1868 by Kirkwood, and are shown in Figure 1, where the number-distribution of asteroids is plotted against the mean distance from the Sun. The gaps occur at certain fractions of Jupiter's period of 11.86 years. This fact indicates that Jupiter has a definite ordering effect upon the asteroidal material. A simple mechanism explaining the Kirkwood gaps is that the gravitational force of Jupiter's large mass drives the asteroids out of the particular resonance orbits; however, the actual mechanical process is apparently more complicated since clustering is observed at some of the expected resonance orbits.

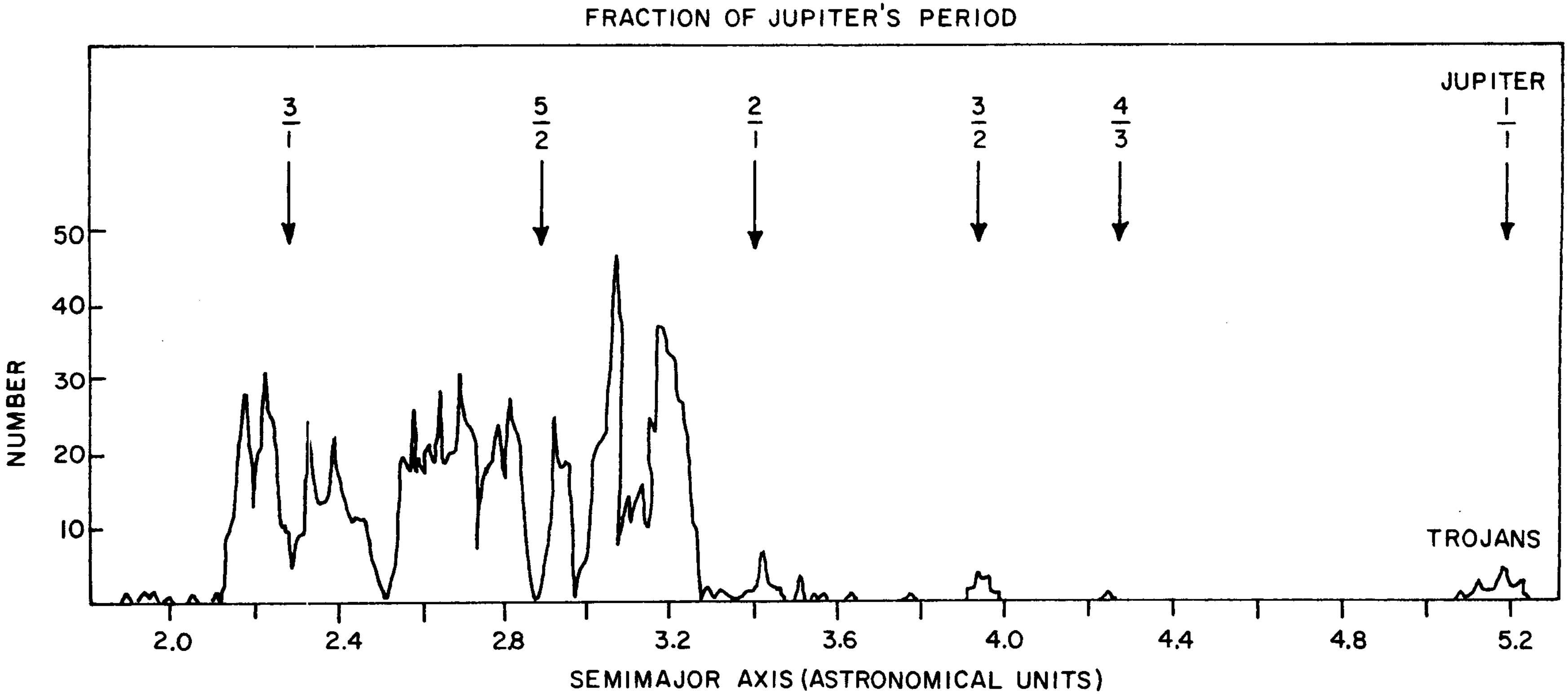

FIGURE 1. THE DISTRIBUTION OF THE ASTEROIDS

As the Kirkwood gaps demonstrate a resonance forcing mechanism, so the twelve Trojan asteroids demonstrate the effects of another type of forcing mechanism also tending to introduce an order into the distribution of the asteroids. In this case, equilibrium points occur where the gravitational forces of the Sun and Jupiter balance the centripetal forces of the orbital motion. As a result, there are two points that rotate about the Sun with Jupiter, the Lagrangian points, about which certain asteroids are observed to cluster. The Trojan asteroids oscillate in closed orbits about these points when viewed from a reference system rotating with the same orbital period as Jupiter. This oscillatory motion is called a libration, and the Lagrangian points are examples of libration points; they are also called stationary points since they correspond to positions of minimum potential energy in a suitable reference coordinate system.

The existence of the Kirkwood gaps and the clustering of the Trojan groups of asteroids are familiar features that have been extensively investigated. However, the significance of these well-known features and of other more obscure properties of the asteroidal distribution present an unresolved problem. The asteroids pose many questions that remain to be answered. Their origins, ages, and material properties are not definitely known. Their role in the origin and evolution of the solar system has not been definitely established, neither has their relationship to other interplanetary matter nor to the planets themselves. Although some asteroids are believed to be the fragments of originally larger bodies, it is not known to what extent the total asteroidal matter consists of primordial condensations or represents the remains of catastrophic events.

Our incomplete knowledge of the asteroids is the result of two fundamental deficiencies: (1) insufficiently developed theoretical methods for the interpretation of the asteroid history based upon current observations, and (2) the lack of adequate observational data concerning the present nature of the asteroids.

2. THE APPLICATION OF ANALYTICAL METHODS

The various methods for the analysis of asteroidal distributions may be categorized into three broad classes:

(1) Celestial mechanics,
(2) Statistical analysis of observational data, and
(3) Ergodic theory.*

The limitations and significance of the methods with respect to particular aspects of the asteroid distribution are outlined below.

2.1 Methods of Celestial Mechanics

The principal tool of celestial mechanics is perturbation theory. This theory has many ramifications but is restricted in its effectiveness by the length of time for

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*The branch of mathematics concerned with finding time averages by means of phase averages (e.g., Birkhoff 1927, or Dunford and Schwartz 1958).

which satisfactory accuracy is attainable. The historical time period can be extended by obtaining more accurate observations of current orbital elements and by improved efficiency in the rate of convergence of the perturbation techniques. Reduction in the accumulation of errors may be achieved by extending the theory to include some of the hitherto neglected forcing terms and by the inclusion of higher order terms. Further improvement is possible if more suitable coordinate systems can be found by means of canonical transformations.

The application of celestial mechanics has led to significant results in (1) distinguishing particular groups of asteroids, (2) the study of resonances leading to clustering or voids, and (3) the determination of those modes of motion involving librations, revolutions with respect to selected rotating coordinate systems, stationary points, and stability of motion.

2.1.1 Asteroid Groups

Within a distinctly recognized group of asteroids, the members are conjectured to have been exposed to some common event. For example, the existence of distinct families was recognized by Hirayama (1918) from an examination of the orbital elements of the known asteroids. The orbital elements considered were the eccentricity, the longitude of the perihelion, the inclination of the plane of the asteroid orbit to the ecliptic plane, and the longitude of the ascending node of the intersection of the asteroid orbit with the ecliptic plane.

These four orbital elements describe the shape and position of the asteroid orbit. The size of the orbit is defined by the semimajor axis which is directly related to the period of revolution. These orbital elements remain constant if no forces except the attractive force of the Sun is acting on the asteroids. However, the perturbing forces of the planets cause the orbital elements to slowly change over the course of thousands of years.

Hirayama considered each orbital element to consist of two parts, the unchanging proper element which characterizes the asteroid's own contribution and a variable part resulting from the perturbing forces of the planets. After determining the proper elements, he discovered five families, the members of each family having nearly the same proper eccentricity and proper inclination, while the other orbital elements are scattered. He concluded that the minor planets belonging to a family are the fragments of an originally larger body broken up by some catastrophic event. At present, many more asteroids have been examined, resulting in extension of the original Hirayama families. Brouwer (1951) lists 29 groups on the basis of some 1723 numbered asteroids in Minor Planets for 1947 published by the Cincinnati Observatory. For this listing, he used improved values for the calculation of the forced motions derived from more accurate planetary masses and including the effects of the principal second-order terms due to the mutual actions of Jupiter and Saturn.

The example of the Hirayama families provides an indication of how groups of asteroids may be selected for examination of originating mechanisms. Kuiper (1950) considered four possible break-up mechanisms: Explosion, rapid rotation, tidal break-up, and collision. He assumed that originally five to ten planets of the size of the largest asteroids were formed between 2 and 3.5 AU from the Sun, and that consequent fragmentation is most likely the result of collisions.

Perhaps more importantly, the method of selection upon which the Hirayama families are based provides an example of how celestial mechanics may be employed to analyze asteroidal distributions and determine possible historical events. The theory applied by Hirayama in deriving the proper elements is only a rough approximation. The approximations may be successful for periods of from 10^6 to 10^7 years but will become meaningless for a billion years. Consequently, if the families are as old as the Earth's crust, about 3 to 5 billion years, the common origin of these asteroids cannot be expected to be found from the theory as applied by Hirayama. Alternatively, if fragmentation occurred after a minor planet had been revolving about the Sun for more than about a billion years, the celestial mechanics technique as applied by Hirayama, perhaps extended and improved, may provide a valid approximation.

It is instructive to examine more closely the celestial mechanics techniques applied to the Hirayama families. Typical restrictions and assumptions generally employed in planetary theory are illustrated. The principal assumptions are the following:

(1) The disturbance function for the perturbing forces of the planets, although not uniformly* convergent, is assumed to be representative of the gravitational force system exerted on the asteroids by the planets.

(2) The higher order terms are assumed to be negligible, although the nature of the retained terms, in reality, depend upon the particular coordinate reference frame chosen.

(3) The effects of the periodic parts are assumed to average out, so that only the remaining secular changes are considered to be significant.

The solution to the differential equations of the resultant dynamical system includes the superposition of the forced motion caused by the planet and the free motion which is the dynamically significant representation of the original initial conditions. It is assumed that the character of this free motion has persisted from the initial formation of the asteroid up to the present time. Hence, families having similar values of the difference between the current observed values and the computed values induced by the planets are assumed to have originated from some common event, and the spread in the values within a given family may serve as an indicator of the corresponding spread in the initial scatter

*The convergence depends upon the historical time interval; the entire series may actually be asymptotically divergent, or semi-convergent (Poincare 1905), while the first terms approximate the disturbance function.

velocities of the fragments.

A more reliable establishment of groups, and especially group ages, requires an extension of the theoretical methods so that some of the restrictive assumptions may be relaxed. Also, the need for more observational information is apparent from Figure 2. Even the most compact Hirayama families show considerable scatter and relatively few numbers. For the estimated minimum of 30,000 asteroids within reach of the large reflecting telescopes, the careful and accurate observation and computation of the orbital elements appears to involve a prohibitive amount of effort. Spacecraft would seem to offer the possibility of observing small asteroids, but it would only be possible to associate a given asteroid with a family after its orbital elements had been determined. This would seem to be an excessively redundant procedure.

The spread in the initial velocity distribution of a fragmented asteroid is correlated with the spread in the proper orbital elements of the resulting family. The initial velocity distribution may be inferred from the application of principles of collision mechanics if the physical impact properties of the asteroidal material is known. To this end, collection and study of asteroidal matter may lead to a sufficiently accurate determination of the required physical properties.

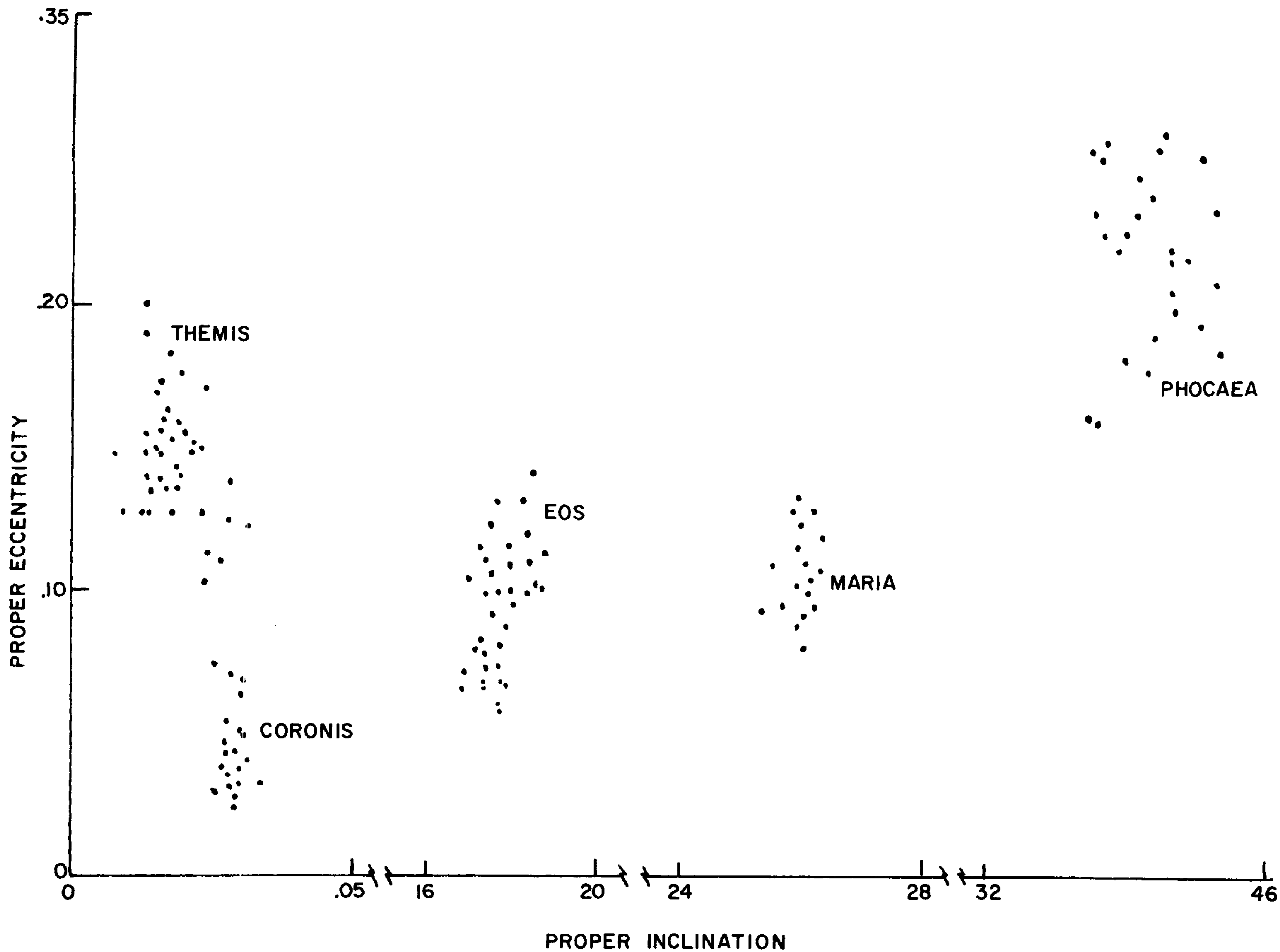


FIGURE 2. DISTRIBUTION OF ASTEROIDS FOR FIVE FAMILIES (BROUWER 1951)

2.1.2 Resonances

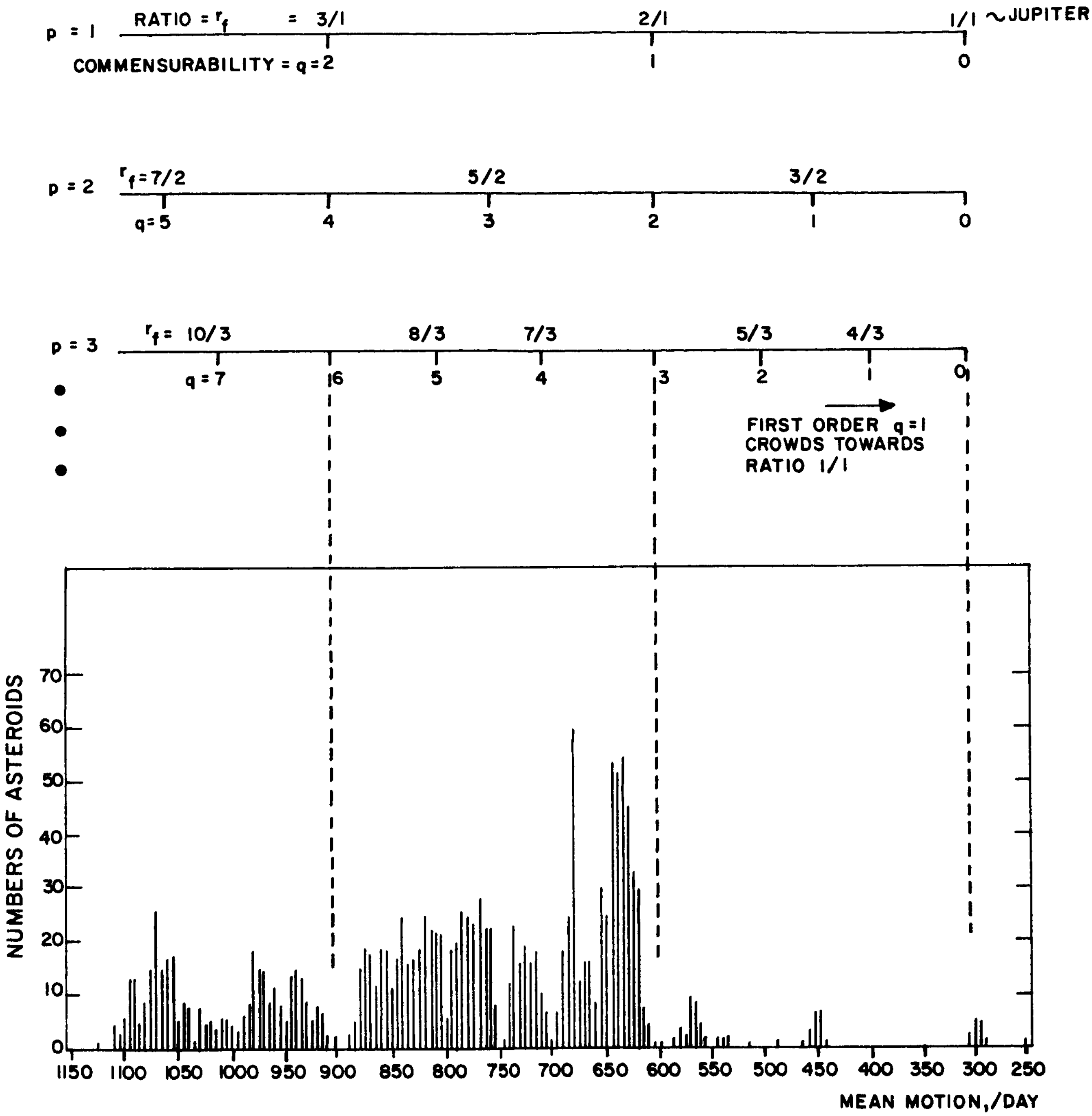
Another aspect of the forcing mechanisms capable of introducing an ordering into the asteroid distribution is manifested by the Kirkwood gaps. The physical conception of an order-producing force mechanism may be clarified by consideration of a contrasting situation where the perturbational forces acting upon an asteroid in orbit are perfectly random. In this example of a non-ordering mechanism, the mean orbit may be expected to be invariant with time since the small effects of the random perturbations tend to average out. On the other hand, if the perturbational forces occur in some definite repetitious manner, the cumulative effect will result in a distinctive trend in the perturbed orbit. Such effects may be broadly described as orbital resonances (Brown and Shook 1933) and may be expected whenever the ratio of the orbital frequency of an asteroid to the orbital frequency of Jupiter occurs as the ratio of integral numbers. In this case, the orbital motion of the asteroid is said to be commensurable with the motion of Jupiter. At such a commensurability, the perturbing forces of Jupiter may be expected to occur with a definite repetition rate, tending to drive the asteroid out of those orbits having periods commensurable with the period of Jupiter. For the commensurable case, the ratio of orbital frequencies can be expressed as:

$$(1 + \frac{q}{p})$$

where q and p are integers, and the value of the integer q is called the "order of the commensurability".

Brouwer (1963) shows in Figure 3 the number distribution of asteroids as a function of their mean angular velocity or motion. The locations of commensurabilities up to order 10 are marked on the upper part of the diagram. The orbital period is directly related to the mean distance of the orbit from the Sun. From the diagram, it may be observed that as the mean radius approaches that of Jupiter, the radii at which commensurabilities of a given order occur tend to crowd together. The orbits in these outer regions of the asteroid belt may oscillate between the closely spaced commensurabilities creating a clustering of observed orbits such as occurs at the 3/2 resonance. The clustering at the 1/1 commensurability on the orbit of Jupiter is the Trojan group and may be explained by the equilibrium of forces corresponding to stationary points of minimum potential energy.

The crowding of commensurabilities for each order as the orbit of Jupiter is approached is to be expected. If the asteroid orbital frequency is expressed in units n of the frequency of Jupiter, then it is well known mathematically that a commensurability will occur for each rational value of n. Since the mean distance from the Sun is a continuum of real numbers, the corresponding spectrum of frequencies is also a continuum. Consequently, there are a denumerably infinite number of commensurabilities for a finite range Δn of



NOTE: RATIO OF ORBITAL FREQUENCIES $= 1 + \frac{q}{p} = r_f$ q, p = INTEGERS
q = ORDER OF COMMENSURABILITY

FIGURE 3. THE KIRKWOOD GAPS AND ORDERS OF COMMENSURABILITIES. (BROUWER 1963)

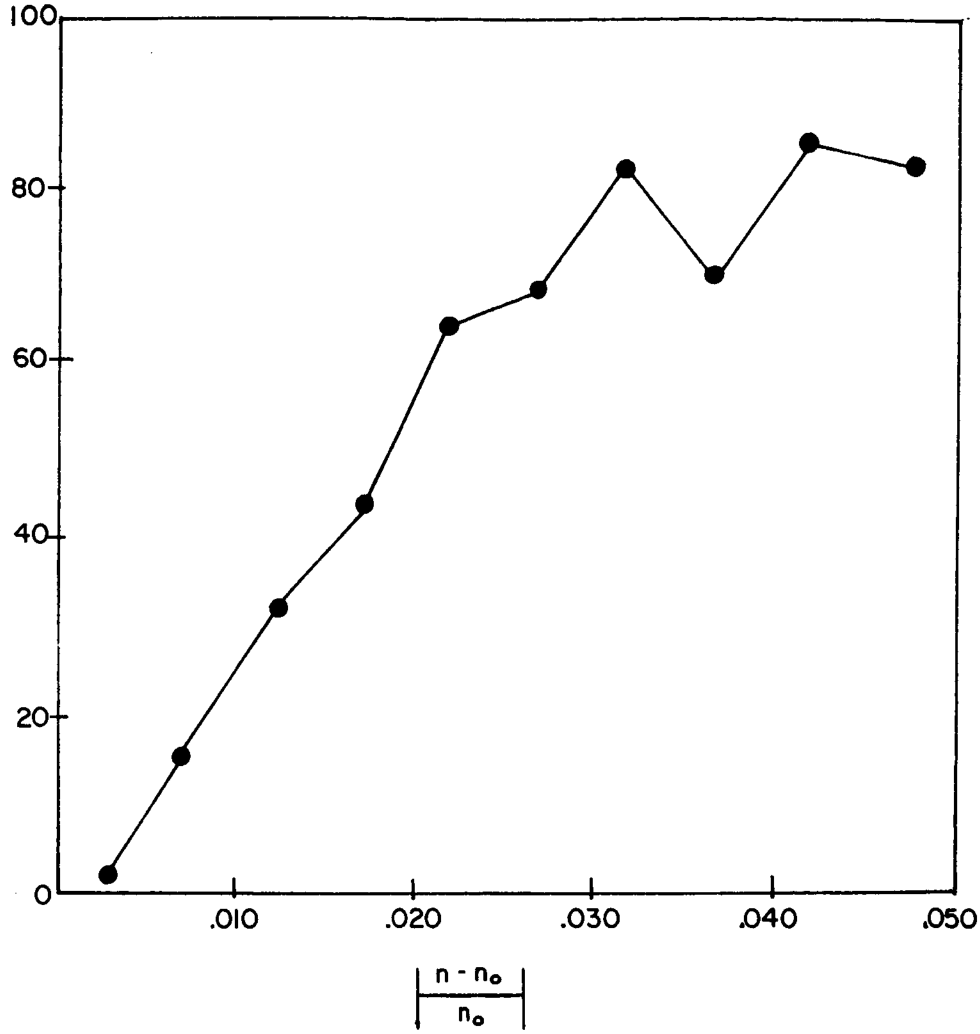
frequencies. This apparently paradoxical mathematical situation can be made physically meaningful if the influence of the perturbing forces diminishes as the order of the corresponding commensurabilities increases. To resolve the apparent ambiguities and explain the appearance of clustering at these outer regions, further observational data and extended analyses are required.

2.1.3 Stationary Points

It was mentioned above that the Trojan group, which clusters at the Lagrangian points, corresponds to positions where the potential energy function has a minimum, or stationary value. From the solution of the differential equations of motion in a suitable rotating coordinate system, the libration orbits of the Trojans about the rotating Lagrangian points may be computed (Rabe 1961). Brown (1933) has analyzed the motions of the Trojans considered as a four-body problem including the perturbation forces of Saturn as well as Jupiter. The principal of a stationary value of a function may be applied to find other distributional features of the asteroids. For example, Hagihara (1957) describes criteria for determining whether the perihelion of an asteroid orbit will oscillate about the perihelion of Jupiter (considered as a stationary point) or make a complete revolution in a reference frame rotating with Jupiter.

Brouwer (1963) has computed the asteroid distributions to be expected in the vicinity of a resonance point (as described above) by demonstrating that in a suitable coordinate system, the correlation of the potential energy, called the Hamiltonian, is an invariant property carried by the asteroid. The distribution of the values of this invariant has a stationary value at a commensurability; and from this fact, the number distribution of the asteroids in the neighborhood of a commensurability is expected to be V-shaped (one leg of the V-shaped distribution is shown in Figure 4).

The analysis, based upon the methods of Hamiltonian mechanics, utilizes special canonical coordinates rather than the coordinates of physical space. For this case, the special coordinates are the modified Delaunay variables, involving dynamical and kinematical parameters of the asteroids. This type of analysis is of importance because the hypotheses introduced into the application of celestial mechanics are clearly brought out and may be tested by observational evidence. The V-shaped number-distributions occur in the neighborhood of resonances in the inner and central regions of the asteroidal ring. At the outer reaches of the asteroidal ring (as the orbit of Jupiter is approached), the radii at which resonances may occur crowd together and complicate the theoretical treatment of orbital stabilities. The asteroidal orbits in these regions may oscillate between the closely spaced commensurabilities creating a clustering. As a consequence, many more detailed data on asteroids are required to extend or modify the existing dynamical theory of perturbational resonances. This is particularly true for the outer parts of the ring. Thus,



$\frac{n - n_o}{n_o}$	COMMENSURABILITY			TOTAL
	2/1	3/1	5/2	NUMBER
.000				
	1	0	1	2
	1	5	10	16
.010				
	1	5	27	33
	5	12	27	44
.020				
	3	24	39	66
	11	24	35	70
.030				
	23	27	34	84
	25	17	29	71
.040				
	21	33	33	87
	26	26	33	85
0.50				
	(117)	(173)	(268)	(558)

FIGURE 4. DISTRIBUTION OF ASTEROIDS IN THE VICINITY OF THREE COMMENSURABILITIES. (BROUWER 1963)

significant advances in the understanding and capability of predicting clustering or voids in the asteroidal distributions may depend upon detailed observations from large reflectors and possibly space probe observations.

2.1.4 Qualitative Theory

In the treatment of the distribution of asteroid orbits by the usual perturbation methods of planetary theory as discussed above, the long-period terms with small coefficients in the disturbing function are ignored. These terms, which are effectively absorbed in the integration constants for a finite interval of time, may very well invalidate the theoretical results if very long epochs are to be considered. Recently, attempts have been made to avoid this difficulty by using methods of the qualitative theory of differential equations.

A theorem important to the problem of asteroidal distributions has been developed by Arnold (1963) based upon topological methods. He states that changes in the orbits due to mutual perturbations remain small for indefinitely long times for the majority of initial conditions for planets of sufficiently small masses with nearly circular orbits in a single plane. The importance of this theorem is that a precise statement is made concerning planetary motions that is not dependent upon the time interval, in contrast to, say, the classical Poisson theorem for the invariability of the semi-major axes, which is restricted to only the lower order terms. The neglected higher order terms become of increasing significance with the passage of time. Applying this theorem to the three-body problem consisting of the Sun, Jupiter, and an asteroid, we can say that there exists a set of initial conditions such that if the initial positions and velocities belong to this set, then the bodies perpetually remain at a limited distance from each other. This theorem is significant not only to the Kirkwood gaps, but also to the underlying hypothesis for the application of ergodic theory where the trajectories are required to be bound to a definite finite region in phase space.

The basic problem of the qualitative theory is to determine the regions to which the asteroid trajectories are bound. These regions can be represented by a multidimensional invariant torus filled by the conditionally periodic trajectories, together with some neighborhood whose extent depends upon the parameters of the system. The motions may be resolved into "fast" and "slow" frequencies. The zeroth approximation is an asteroid trajectory described by the Keplerian orbit with constant values of the orbital parameters. The first approximation permits small oscillations of these parameters, and the second approximation permits secular changes in the eccentricity and longitudes of the ascending node and perihelion and is called the Lagrangian motion. The Keplerian motion corresponds to a fast frequency and the Lagrangian motion to a slow frequency that is conditionally periodic. A motion is called conditionally periodic if it returns arbitrarily close to any previous point.

Two possible classes of motions are librations and rotations, which form two sets of topologically irreducible curves on the torus. By a suitable canonical transformation, a canonical "angle" variable w may be found such that:

$$q(w + 1) = \begin{cases} q(w), & \text{libration} \\ q(w) + 2\pi, & \text{rotation} \end{cases}$$

If the parameters of the asteroid system are slowly varying such that in the limit the time derivatives of the parameters go to zero, then the system is said to undergo an "adiabatic change".

The purpose of this discussion of qualitative methods is to point out the existence of this point of view and the possibility that through the use of these techniques, namely, adiabatic invariants, invariant tori, and ergodic theory, questions concerning asteroidal distributions may be resolved. The questions are essentially those of stability of motion and the determination of the extent of regions to which the trajectories are bound for infinite periods of time. This is in contrast to the general aim of perturbation techniques to determine ephemerides-type information predicting the particular spatial history for individual asteroids. When the actual historical trajectory, albeit approximate, cannot be computed, useful information can still be derived from qualitative theory. From such information, predictions can be made of trends toward clustering or dispersion, and corresponding rates, from which direct inferences may be derived concerning origins and evolutions.

2.2 Statistical Analysis of Observational Data

The disadvantages of the classical celestial mechanics methods include mathematical complexity and limitations of the historical time period, with the advantage that relatively few observational data are required. By obtaining large samples of significant data, much mathematics, often of unmanageable complexity, may be avoided by the application of statistical methods. Through the examination of such data, attempts have been made to discover relationships upon which to base empirical laws or conjectural hypotheses. By this procedure, heuristic theories have evolved regarding planetary origins and the recognition of families of asteroids with common characteristics.

2.2.1 Initial Scatter Velocities

An example of the application of a statistical examination of data combined with celestial mechanics is Brouwer's (1961) analysis of the average velocities with which fragments are dispersed from a fractured parent body. He concluded that the average dispersion velocities need only have been about 300 fps for the more compact families, and about 1000 fps for the Flora family. On the basis of perturbation theory, the first order secular changes of the orbital elements are directly related to changes in the initial velocities. From the spread in A,B, $(\pi_1 + \theta_1)$, and a, the corresponding spread in the scatter velocities can be computed. The results are shown in Table 1, where A and B are the proper eccentricity and

Table 1

MEAN VALUES AND STANDARD DEVIATIONS FOR FAMILIES

(Brouwer 1951)

Group	No.	a	$\overline{A}$	$\overline{B}$	$\Delta \dot{}$	$\Delta \dot{r}$ km/sec	$\Delta \dot{\zeta}$
1	53	3.1367	.1550	.0239	.455	.071	.100
2	58	3.0149	.0747	.1757	.335	.018	.116
3	33	2.8753	.0491	.0371	.159	.055	.040
4	17	2.5460	.0991	.2596	.692	.039	.108
5	21	2.3653	.2413	.4015	1.275	.218	.692
6	23	2.2175	.1253	.0511	.573	.110	.221
7	62	2.2182	.1368	.0801	.695	.097	.220
8	9	2.2167	.1448	.0976	.269	.081	.070
9	31	2.2528	.1582	.1129	.498	.163	.247
11	4	3.3879	.1217	.3659	1.032	.023	.412
12	6	3.0617	.1164	.2116	.643	.066	.430
13	7	2.9617	.0563	.0521	1.127	.105	.303
14	8	2.9757	.1908	.1954	1.159	.104	.318
15	6	2.8656	.1536	.2085	.413	.001	.311
16	8	2.7832	.1570	.1566	1.583	.013	.315
17	10	2.7482	.2681	.2795	1.003	.060	.485
18	10	2.6749	.1954	.2663	1.040	.067	.162
19	7	2.6729	.1729	.1973	.485	.046	.090
20	8	2.6369	.1306	.2266	.029	.080	.123
21	6	2.6253	.1905	.2463	.325	.047	.354
22	8	2.5725	.1495	.1796	.553	.061	.370
23	5	2.5502	.2492	.1112	1.519	.034	.329
24	9	2.4471	.1643	.0532	.366	.090	.089
25	11	2.4355	.1574	.0873	.332	.142	.153
26	8	2.4341	.0592	.0507	.595	.077	.403
27	8	2.3683	.1882	.1862	.764	.028	.422
28	6	2.7163	.2519	.5532	1.808	.242	.401
29	7	2.5969	.3424	.2942	2.380	.093	.361

inclination, $(\pi_1 + \theta_1)$ is the sum of the proper perihelion and node, which should be time independent,* and a is the semi-major axis. The orthogonal velocity components are $(\dot{\xi}, \dot{\eta}, \dot{\zeta})$ where $\dot{\xi}$ is normal to the orbital plane, and $\dot{\eta}$ is tangent to the orbit.

The standard deviations in $\dot{\xi}$ are on the average greater than $\dot{\eta}$ by a factor of 6, and greater than $\dot{\zeta}$ by a factor of 2.7. Before inferences can be derived from these deviations in the scatter velocities, more should be known about the expected scatter velocity distribution for a given breakup phenomena. Also, more should be known of the effect of secular variations on the spread of the initial scatter velocities and the ratios between the three components. Since the semimajor axes are known to be more stable than the inclinations, eccentricities, and nodes, the standard deviations in $\dot{\eta}$ should be expected to give the best indication of the scatter velocities. If the cause of breakup were a collision, then from comparison of $\Delta \dot{\eta}$ with the expected collision velocity, some inferences could be derived about the relative sizes of the colliding bodies. For example, if the expected collision velocity were 1 km/sec then since the standard deviations of $\dot{\eta}$ are much smaller than this value, the fragmentation is likely the result of a collision of the parent body with another body of much smaller mass.

*This is true because the theory of Hirayama indicates that within a given group the rate of advance of the perihelion should equal the rate of regression of the node.

2.2.2 Asteroidal Grinding and Group Ages

From an examination of the size distribution of the asteroids, Anders (1965) has made the following tentative hypotheses:

(1) "The asteroid belt is not in a highly fragmented state. The present distribution is only a few steps removed from the original one."

(2) "The collisional half-life of asteroids, for a loss of one-half their mass, is 6.1×10^9 yrs."

(3) "The cross-sectional area of all asteroids brighter than magnitude 14 has increased by only 6 percent during the last 4.5×10^9 yrs."

(4) "The average crushing strength of asteroids seems to be about 2×10^8 dyne/cm^2."

(5) "The half-life for dispersal or destruction of Hirayama families is greater or equal to 2.2×10^9 yrs."

The size distribution of the asteroids on the basis of absolute visual magnitude g is shown in Figure 5 by Kuiper (1958), who has conjectured that the change in slope of the curve at magnitude g = 9, separates the asteroids into two classes. The larger asteroids are essentially the primeval bodies, and the smaller asteroids (g greater than 9) are fragments from collisions.

On the basis of the observed size distribution, various semi-empirical laws have been proposed to represent the gross collisional processes and predict the size distributions down through the smaller unobserved asteroids. Hawkins (1960) has

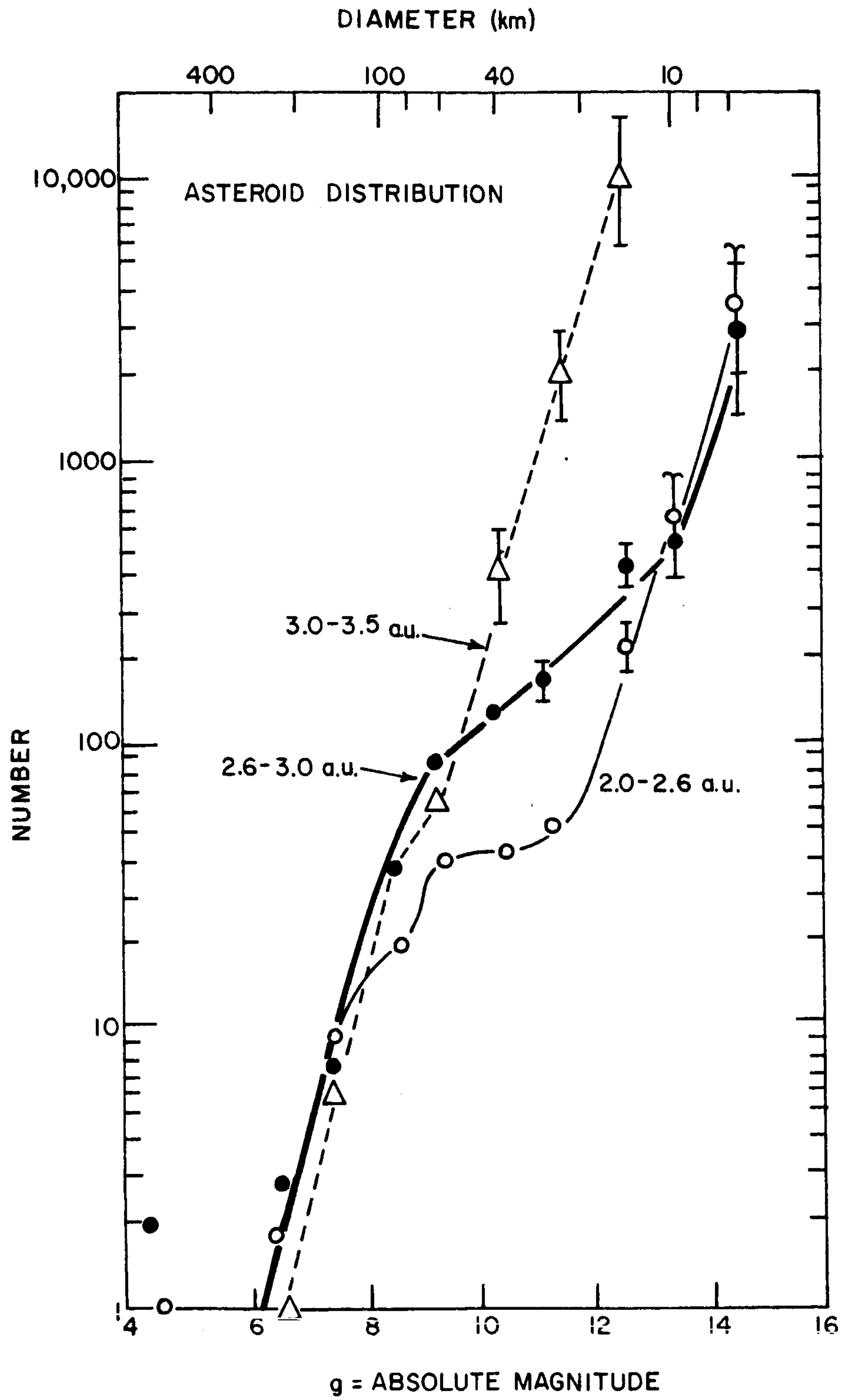


FIGURE 5. ASTEROID SIZE DISTRIBUTION IN THREE ZONES OF THE ASTEROID RING (KUIPER et al., 1958)

proposed a "comminution law":

$$\log p = k \log x + \text{const}$$

where Δp is the fraction of the total mass in the size range $(x, x + \Delta x)$. For early grinding, it is assumed that $k = 1$, and as grinding continues, the value of k approaches zero. The original application of the law was for the grinding of quartz in a rod mill (Gaudin 1944).

The hypotheses of Anders, mentioned above, are based upon a population growth law, in which the collision rate is assumed proportional to the number of asteroids present. Jaschek and Jaschek (1963) assume a collision probability law proportional to the square of the number n of asteroids:

$$\frac{dn}{dt} = k\, q\, n^2$$

where q is the increase in the number of bodies due to collision and $k\, n^2$ is the collision frequency. They arrive at an estimate of about 5×10^6 yrs for the mean family life. It may be noted that age determinations of iron meteorites indicate major collisions at times 630×10^6 and 900×10^6 yrs ago; and hypersthene chondrites indicate a major collision 400×10^6 yrs ago, with secondary collisions in a wide range over the past 60×10^6 yrs.

If the age estimates of the asteroid families based upon celestial mechanics or collision processes were more reliable, perhaps some significant correlations with the age indications of asteroidal meteorites could be made.

2.2.3 Asteroid Origins Based Upon Observations

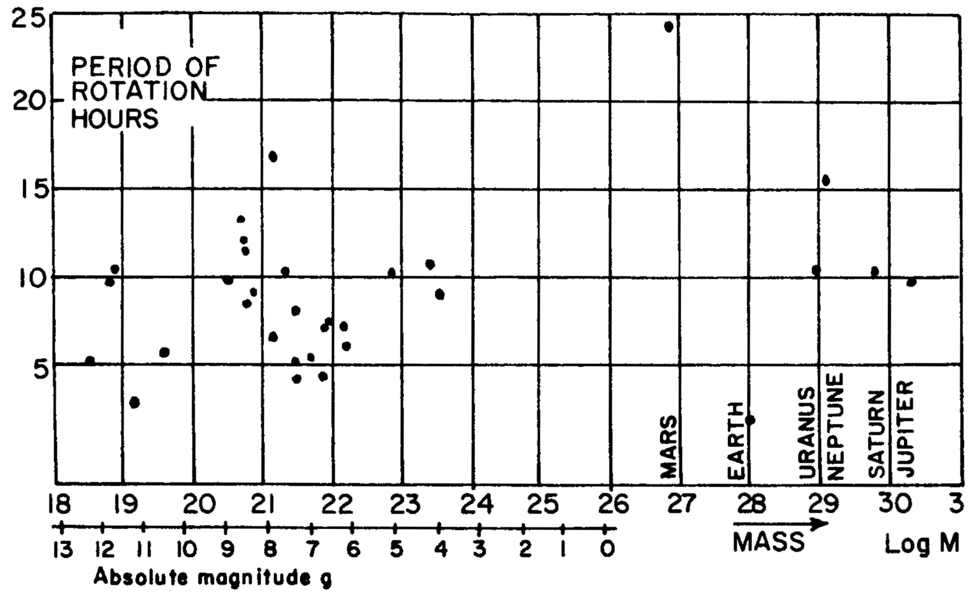
The discovery of the first asteroid (Ceres) was thought to be the planet between Mars and Jupiter as predicted in accordance with Bode's law. The later discovery of Pallas (1802) was unexpected, and subsequent discoveries of Juno and Vesta led immediately to the belief that these minor planets were the fragments of the predicted planet which had broken up (Olbers' theory). The conjecture that most asteroids are fragments of primeval planets is still considered tenable; and observations of what appear to be broken surfaces, particularly the irregular shape of Eros, tend to support this hypothesis. About two dozen asteroids indicate variations in brightness greater than 26 percent with axial rotation periods of a few hours. Kuiper (1953) has concluded that collision is the most likely cause of the breakup, rather than explosion, rapid rotation, or tidal forces. He suggests there may originally have been 5 to 10 small planets. Ceres, for example, may be an original condensation, as indicated by the fact that the light from this planet and from Pallas are of almost constant magnitude. The original probability of a collision is estimated to be of the order of 0.1 in the past 3×10^9 yrs for orbital inclinations below 5° and eccentricities less than 0.1, considering an average collision grazing velocity of about 1 km/sec.

The probability of secondary collisions is much higher, and, for example, the Hirayama families are due to relatively recent secondary collisions, some perhaps of ages in the order of 2 to 3 x 10^6 yrs. The remaining 80 percent of the asteroids (non-Hirayama families) may have been formed during earlier collisions. The increasing fractionation by secondary collisions is indicated by the large number of meteorites which appear to be of asteroidal origin and have led Orlov to estimate over a million asteroids of size greater than 2 km, while Hubble and Baade, on the basis of 100-inch telescope photographs, estimate less than 50,000 are of apparent magnitude below 19.

An alternative to the hypothesis that the asteroids are fragments of a few large bodies formed directly by condensing plasma is that the asteroids are planetesimals which are the products of the initial stages of plasma condensations and, for some reason, have terminated the process of further agglomeration into larger bodies.

The rotation of the asteroids, which permitted the detection of irregular surfaces by variations in the brightness, is used by Alfven (1964) to support an argument that the asteroids are not mainly collision fragments. The exchange of rotational energies upon collision should be expected to be somewhat equal, so that the more massive bodies would have smaller angular velocities. However, no such relation between angular velocities and asteroid size is indicated as shown in Figure 6.

The angular speeds are far below the rotational stability limit, so it is unlikely that breakup is the result of



PERIOD OF AXIAL ROTATION FOR THE ASTEROIDS IN RELATION TO THEIR MASSES. (ALFVÉN, 1964)

Table 2

PERIODS OF ASTEROIDS (Alfven 1964)

No.	Name	Abs. Mag.	Period (hours)	No.	Name	Abs. Mag.	Period (hours)
1	Ceres	4.0	9.08	22	Kalliope	7.4	4.07
2	Pallas	5.1	10.12	25	Phocaea	9.0	9.95
3	Juno	6.3	7.22	27	Euterpe	8.5	8.50
4	Vesta	4.2	10.68	30	Urania	8.7	13.67
5	Astraea	7.9	16.81	39	Laetitia	7.3	5.14
6	Hebe	6.6	7.28	40	Harmonia	8.4	9.13
7	Iris	6.7	7.13	44	Nysa	7.9	6.42
9	Metis	7.2	5.06	61	Danae	8.6	11.45
11	Parthenope	7.7	10.7	321	Florentina	11.3	2.87
15	Eunomia	6.2	6.08	433	Eros	12.3	5.27
16	Psyche	6.8	4.30	511	Davida	7.0	5.17
17	Thetis	8.6	12.27	753	Tiflis	11.8	9.84
20	Massalia	7.4	8.10	984	Gretia	10.6	5.76
				---	PH 1931	11.7?	10.23

FIGURE 6

centrifugal forces; and the relative constancy of these speeds is comparable to those of the major planets. In the diagram shown, the Earth period is estimated as 2.6 hr, a value suggested prior to capture of the moon; and for the uncertain periods of Venus and Mercury, it is assumed that their axial rotational periods coincide with their orbital periods as a result of solar tides. The fact that the periods of the planets, both major and minor, are similar points towards a common formation mechanism involving the same rotational period of about 10 hr.

In support of the latter hypothesis, that the structure of the asteroid distribution is not due to collisional processes, but is rather the product of the original evolution by condensation processes in a plasma, Alfven (1954), has compared the asteroid ring with the rings of Saturn. On the basis of his analysis, the orbits of the condensation grains should intersect the equatorial plane of the central body at 2/3 the original radius of condensation where they will be captured by collisions and move in nearly circular orbits. A similar process may have occurred in the rings of Saturn. The Cassini division between the A and R rings at the 1/2 resonance with Mimas corresponds to the Kirkwood gap in the asteroid ring at the same resonance with Jupiter. Also to be considered in the mechanism of formation of Saturn's rings is Roche's limit (Jeffreys 1947a) within which tidal forces predominate and prevent growth due to accretion.

2.3 Ergodic Theory

Because of the inadequacy of classical celestial mechanics to provide an explicit solution for the asteroid motions over arbitrarily large times, methods of analysis should be sought other than those attempting to construct integral curves from the differential equations of motion. Knowledge of the early history of an asteroid trajectory may contribute to the formation of cosmogonic hypotheses. For example, it would be useful to be able to trace back from presently observed motions to find the last time, if ever, two asteroids have simultaneously occupied the same position. The time interval measured from the instant of this common origin to the present may be defined as the orbit lifetime of the two asteroids.

If the asteroid lifetime is too great to be solved deterministically because of divergence of the analytical methods, then recourse can be made to the application of ergodic theory. An attempt of this kind has been made by Milder (1963) to a problem of orbital collision probabilities applied to the lunar impact of a satellite. For this problem he used the ensemble averages of statistical mechanics. For application to the problem of asteroid collisions, the orbits vary secularly and can be considered to generate an ensemble of almost Keplerian orbits. For a set of asteroids, this ensemble provides a sufficiently large number of orbits to permit a statistically significant interpretation. By the introduction of an ensemble measure into the space occupied by the asteroid trajectories, a meaningful probability occupation density can be defined. The event of a simultaneous occupation of a region by two asteroids is referred to as a "collision", and an average frequency of such collisions for an ensemble of secularly varying orbits provides a basis for defining an expected, or most probable, orbit lifetime.

We have developed a method which treats each individual asteroid, considered to have an insignificant mass, as an element of a three-body system consisting of the asteroid, the Sun, and Jupiter. A simplification is obtained by considering the orbit of Jupiter to be circular, but this assumption is not essential. The resulting asteroid orbits are bound to finite regions in a generalized phase space, the configuration-momentum space, and each asteroid is bound to a surface of constant energy in this phase space. By the employment of suitable ergodic theorems, the average expected lifetimes of all the observable asteroids may be computed. If these computations are done, then from a number-frequency plot of the asteroids versus lifetimes, the clustering of asteroids at distinct points will provide a criterion for forming groups having probable common origins. In addition to providing a basis for the formation of dynamically significant groups, this method might yield a direct estimate of the group age.

3. RECOMMENDED OBSERVATIONS

Together with consideration of analytical methods for the interpretation and prediction of the asteroid distribution, attention should be directed to questions concerning possible observations. Our present knowledge of the asteroids has been gained primarily from observations by remote sensing of the position, size, state of motion, and corresponding number-distributions. In the future, we can also expect new data to be derived from remote observations; however, significant information can be obtained from the actual collection of asteroidal material for examination of its structure and composition. Both methods of obtaining observational data are implied in the following list of recommended observations. In the listing the relationship of the observations to theoretical concepts is discussed.

3.1 The Number and Size Distribution of Asteroids

Although most of the asteroids of photographic magnitude brighter than 14 at opposition are probably known, the number distribution of asteroids down through a continuous distribution of sizes is not. Because of the probable multitude of smaller asteroidal objects, a complete survey would be impractical; however, a detailed set of observations, including the smaller size bodies, made over a particular region of interest, would provide data significant to the refinement of celestial mechanics methods. Of particular interest to the analysis of stability is the number distribution in the vicinity of libration points. Also, although not strictly a spatial neighborhood, detailed observations are required in the vicinity of a Kirkwood gap with particular emphasis on the outer part of the asteroid ring.

When a Kirkwood gap occurs at a value of the semimajor axis of the asteroid orbit such that the orbital period is exactly commensurable with that of Jupiter, then an approximate celestial mechanics theory predicts a V-shaped number distribution in the neighborhood of the gap. Detailed observations are required to verify this theory. At the outer reaches of the asteroid ring where the smaller asteroids are not observable from Earth, a crowding of commensurabilities occurs. Additional data in this region are required to resolve ambiguities involving the apparent clustering of orbits observed at these outer radii rather than the expected gaps, and to determine the extent of the influence of the various orders of the commensurabilities.

Observations of the size distribution through the smaller sizes not presently observable from Earth would also test hypothetical comminution laws for grinding or accretion and establish the exponents for existing empirical power laws and exponential growth laws (e.g., Hawkins 1960, Jaschek and Jaschek 1963, Anders 1965).

For the type of collision discussed above, the mutual gravitational force between the colliding bodies is assumed to be negligible. This assumption is not valid for encounters of asteroids with planets, where major orbital deflections may occur without actual physical contact. For this type of encounter, there is no definite trend to reduce the orbital inclination. The statistics of asteroid encounters with planets has been treated in detail by Opik (1951, 1963).

3.2 Particle Distribution Out of the Ecliptic Plane

The distribution of smaller asteroidal particles out of the ecliptic plane is almost unknown. A knowledge of this distribution would permit evaluation of evolution hypotheses involving condensation processes. In particular, a theoretical treatment by Alfven (1952) indicates that original orbits should be produced with eccentricities of about 0.33. Correlation with in-plane distributions at corresponding radii will indicate the extent that collisional processes reduce radial velocities, and consequently the eccentricity, and also decrease the out-of-plane velocity components, bringing the orbits into the ecliptic plane. Since collisions are non-conservative, the relative velocities are reduced by friction and imperfect restitution. From the consequent reduction of the relative velocities, a reduction in the spread of orbital inclinations should be expected with a corresponding increased collisional frequency. An analogous mechanism in certain respects may have occurred in the formation of Saturn's rings which also involved an outward transfer of angular momentum from the inner ring particles of higher angular velocity, tending to spread the particles in a thin layer (Jeffreys 1947b).

Correlation of asteroidal inclinations with the spread of dust about the ecliptic plane indicated by the broadening of zodiacal isophotes (Fesenkov 1959) should be attempted; and computations can be made relating the relative velocities of ejection of eroding dust particles originating on the surface of an asteroid, the orbital inclination, and the dispersion of the ejected particles.

From the observed distribution of higher inclination orbits, the poorly convergent perturbation solutions for the secular variations of orbital elements may be tested. Of particular interest is the motion of the perihelion, for which tentative criteria at present indicate conditions for the existence of stationary points. Improved understanding of the motion of the line of apsides for higher inclination orbits can be extended to the determination of the orbital motions of comets and better understanding of the relationship between comets and meteor streams (Hamid 1950).

3.3 Detailed Motions of Individual Asteroids

Based upon current observations of the orbital elements of an asteroid and a knowledge of the perturbing forces, the path history over which the asteroid has traveled may be computed. However, the errors in the initial observations are reflected in errors in the computed path which magnify as the computation proceeds back through historical time. Although an unwarranted amount of labor would perhaps be involved in attempting to improve the accuracy of all currently observed asteroid data, the selection of particular groups or regions for highly precise measurements could reveal significant historical features.

For example, the perturbation methods forming the basis of the Hirayama families are limited to periods of the order of 10^6 to 10^7 years. With a somewhat more complete theory and more accurate values of the orbital elements, the time interval in some cases could, perhaps, be extended back to the actual intersection of the secularly varying orbits.

For the analysis of the generation and dispersion of the asteroid families, combinations of statistical and celestial mechanics techniques have been employed. Sufficient knowledge of the smaller members of many of the recognized families is lacking for the provision of a satisfactory statistical model. From the spread in the scatter velocities of a larger sample than is now available, estimates of the ages of families can be computed, and an improved understanding of the mechanism of the perturbing forces may be derived. Also, this knowledge will be of value in understanding some of the outstanding questions regarding the motions of the planets. In particular, there remain questions of orbital stability (e.g., Hagihara 1957), and more accurate estimates are required of the masses of those planets not having satellites.

Accurate measurements of axial rotations and orientations over a range of asteroid sizes will provide evidence indicating whether the bodies are collision fragments or are the direct products of an original primordial condensation.

Alfven (1964) has suggested that over a series of collisions, an exchange of energies would tend to equalize rotational energies so that the more massive bodies will have slower angular velocities than the smaller fragments. On the other hand, the existence of a group of asteroids of widely varying sizes having about the same rotational speeds would indicate some common evolutionary process.

3.4 Shape and Surface Conditions

Obviously an accurate knowledge of surface reflectivity would permit an improved correlation of the numerous Earth-based data involving observed brightness with the asteroid size. A fairly symmetric shape with uniform surface conditions would argue against the probability that the asteroid is a collision fragment, as would be indicated by an irregular shape and a jagged surface contour. Observations of erosion and craters (e.g., Witting et al. 1965) would indicate relative age and reveal the history of impacts with other interplanetary debris. Observations of this nature would be particularly helpful in providing needed information on collision frequencies.

3.5 Structural and Chemical Composition

A detailed knowledge of the physical properties of asteroidal material would be of obvious direct value in determining ages, collisional history, and evolutionary processes. Also, comparison could be made with Earth intercepted meteorites to establish origins and to determine the now unknown alterations in an original meteorite caused by its passage through the atmosphere. Of considerable interest would be the comparison of physical properties with a comet core, which could serve to resolve such questions as whether the comet cloud, if one exists, is of asteroidal origin, perhaps retaining the original icy-conglomerate coating, or whether the Apollo group of asteroids is composed of old comets.

Knowledge of the structural strength properties may provide a key to the selection of parameters for the collision mechanics. The impact strength is required in order to estimate the minimum impulse required for fractionation, and from this knowledge improved estimates of the relationship of critical collision velocity and impacting mass may be made. From the more detailed physical properties, the number and physical state of the fragments can be computed.

The physical and chemical properties of recovered meteorites (Anders 1963) may provide clues to the complicated asteroidal history. There are four recognized types of meteorites: the "irons" consisting chiefly of nickel-iron alloys, the stony irons with the additional inclusion of silicate minerals, the "stones" consisting largely of silicates comprising the chondrites which contain small globules of silicate minerals, and the achondrites which are free from chondrules and nickel-iron and may contain plagioclase and sometimes approach a basaltic composition. Inferences based upon the structural details of the meteorites are as yet ambiguous and inconclusive in deriving originating mechanisms.

The occasional presence of diamonds and the crystal structure of the nickel-iron alloys, if not created by high energy of a collisional impact, point towards originating conditions of high temperature and pressure as might be expected within a small planet. On the other hand, some of the stones have pyroclast-type structures and sometimes contain fragments bearing high-pressure materials mixed with chondrules, indicating a fluidization mechanism, low-pressure volcanic conditions, and rapid cooling.

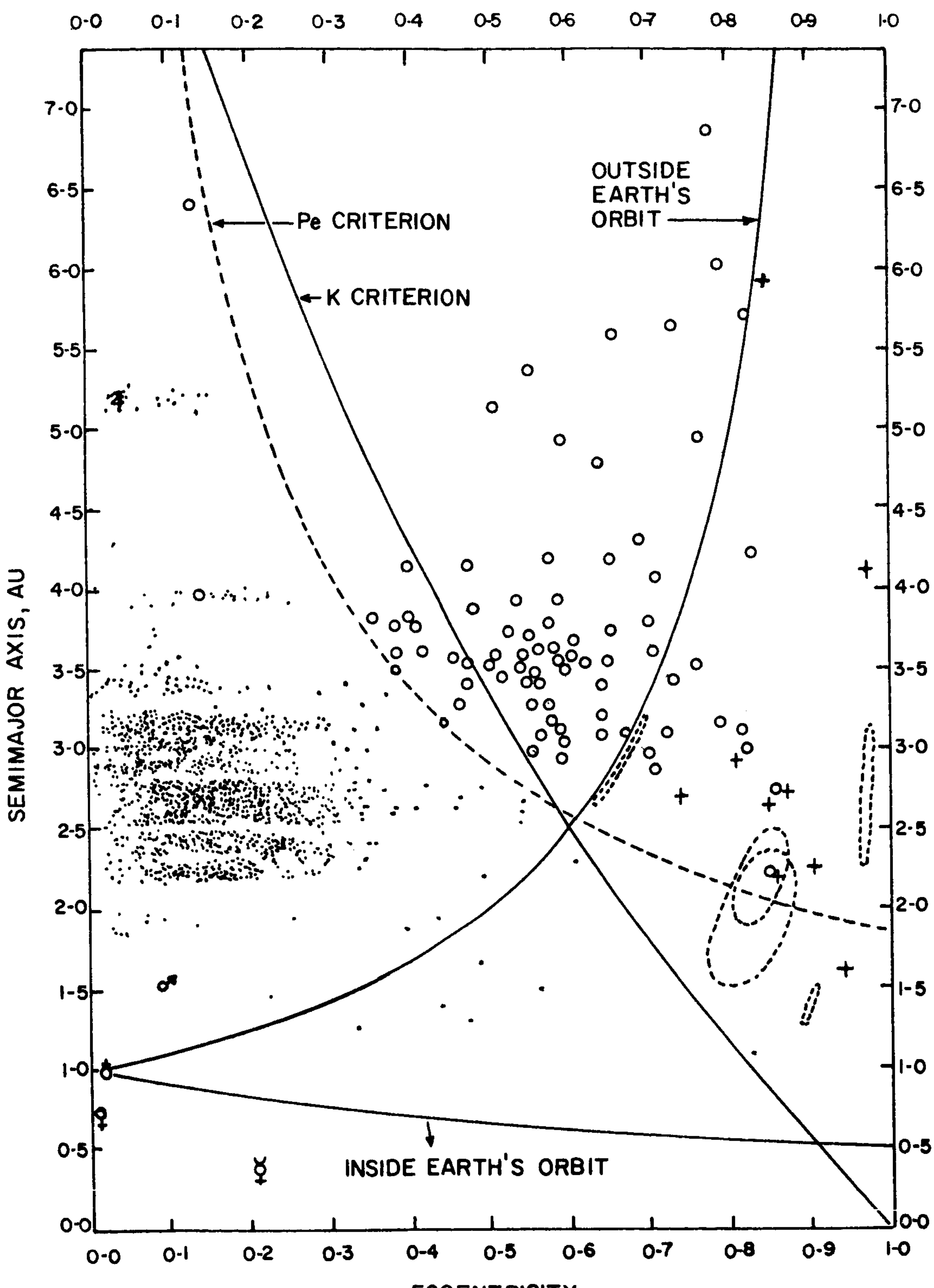
Age estimates of the meteorites are based upon the decay rates of radioactive elements. From the time of the crystallation of the minerals, the lead isotopes 206, 207 and 208 steadily increase in abundance while the isotope Pb^{204} remains unchanged in amount. The constitution of the primeval lead can be determined from the iron meteorites which contain only minute fractions of uranium and thorium. For the stony meteorites, however, considerable radiogenic lead has been created in their silicate minerals. The time of formation of the radiogenic lead can be determined by subtracting the Pb^{204} from the other isotopes and forming the ratio Pb^{207}/Pb^{206}. For samples of the same age, the points plotted on a Pb^{207}/Pb^{204} versus Pb^{206}/Pb^{204} graph should fall on a straight line. Such isochrones for various stony meteorites should intersect at a common point representing the primeval lead of the iron meteorite.

More complicated and uncertain is the estimated evolutionary dating of collisions based upon interpretations of isotope changes resulting from the action of cosmic rays upon the newly exposed surfaces in comparison with the original surfaces.

The question on the extent to which observed meteors represent asteroidal rather than cometary material has not yet been fully established. Criteria have been developed to indicate the origins of meteoric bodies from their orbital elements. The apparent inclination i, eccentricity e, and semimajor axis a of an entering meteoroid can be determined from the analysis of meteor photographs. Based upon the number frequency plot in the a, e diagram, criteria have been suggested for distinguishing asteroidal and cometary meteoroids; for example, the K-criterion of Whipple (1954)* or the Pe-criterion which requires that $ea^{3/2} < 2.5$ for asteroidal origins. Both of these criteria are shown in Figure 7 by Kresak (1965). However, through collisions, many asteroidal particles may be made to appear as if they were of cometary origin. Consequently, an improved understanding of the collisional processes with ancillary observations of asteroid frequency distributions are necessary to establish the true origins of many ambiguous meteors.

Although a considerable increase in eccentricity is required to cause a circular orbit at a radius of 4 AU to come within reach of the Earth orbit, the smaller collisional particles may be brought into the a(1-e) distance by the Poynting-Robertson radiation drag. Particularly desirable

*Requires that $\log (a(1+e)/(1-e)) < 1$ for asteroidal origin of meteorite.



•-ASTEROID, o-COMETS, +-MEAN ORBITS OF METEOR STREAMS, GREEK SYMBOLS-MAJOR PLANETS, DASHED OVALS-DISPERSION OF MAJOR METEOR STREAMS.

FIGURE 7. CRITERIA FOR METEOR DISCRIMINATION OF ASTEROIDS (KRESAK 1965)

are observations of the Apollo group, which is sufficiently close to the Earth orbit to provide meteoric particles but is most difficult to interpret. This group does not constitute a continuous distribution of orbits from the main asteroid belt and is of greater complexity because of the combined significant perturbation effects of both Mars and Jupiter.

4. CONCLUSIONS

A limited understanding of some of the well-known features of asteroidal distributions has been gained through the methods of celestial mechanics. For example, certain aspects of the Kirkwood gaps, the Hirayama families, and the Trojan clusters have been analyzed and explained, but there remain important questions to be resolved. Examination of the literature of asteroid distributions leads to the conclusion that the general, long-term motions of the asteroids are at present imperfectly understood. A deeper understanding of the features of the distribution of asteroidal material will provide clues to the origins of the solar system and explanations of related problems concerning interplanetary matter.

Analyses based upon examination of observational data are presently at a rudimentary stage, and any major progress is dependent upon further observational data. Mathematical and mechanical analyses have yet to provide definitive answers to the outstanding questions. However, modern developments in qualitative and topological theories of differential equations and ergodic theory have not yet been fully utilized in the dynamical analysis of planetary motions and are capable of providing progress in the interpretation of asteroidal distributions. High-speed machine computations may permit numerical analyses valid sufficiently far back in time to determine collisional origins if sufficiently accurate input data is available. Further observational data, such as may be provided by spacecraft, are required to extend and support cosmogonical theories involving asteroidal matter. A summary list of recommended observations is shown in Table 3.

Table 3

RECOMMENDED OBSERVATIONS

Observation	Specific Requirements	Purpose
Number and size distribution (in ecliptic plane)	1) In neighborhood of a libration point	Determine possible clustering
		Test hypotheses for stability analyses
	2) In neighborhood of a resonance with Jupiter period	
	a) Inner & central regions of asteroid ring	Test hypotheses and methods of celestial mechanics that now predict V-shaped number distribution
	b) Outer regions of ring	Determine if clustering occurs and the extent of influence of higher order commensurabilities
	3) Through a continuous range of smaller sizes	Test comminution laws and establish semiempirical coefficients for fragmentation processes
Distributions out of the ecliptic plane	1) Orbital eccentricities	Test evolution hypotheses regarding plasma condensation processes
	2) Orbital inclinations	Test poorly convergent perturbation techniques for secular variations of higher inclination orbital elements
		Determine dispersion and relative velocities of eroding particles from asteroid surfaces
	3) Motion of perihelion	Understand motion of apse line for higher inclination orbits; also of significance to motion of comet orbits and relation to meteor streams
Orbital elements	1) For smaller members of recognized families	Provide a more adequate statistical model
		Correlate spread in parametric values with the initial collision scatter velocities to provide estimates of family age
		Permit determination of proper values for families

Table 3 (Cont'd)

Observation	Specific Requirements	Purpose
	2) More accurate measurements of members of a selected family	Extend validity of computations of secular variations over greater time interval
Orbital directions		Provide evidence relevant to cometary origins
Axial rotation and direction		Evidence of origins: collision fragments or primordial condensations
Surface physical condition	1) Reflectivity	Correlation with Earth-based observations and uniformity indicates non-fragment
	2) Roughness	Indicates collision fragments
	3) Erosion	Age indicator
	4) Craters	Impact history
Material physical and chemical properties		Provide impact parameters for collision mechanics
		Comparison with recovered meteorites to establish meteorite origins
		Comparison with comet core
		Age estimates
		Collision history

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